

Accessing Quark Orbital Angular Momentum through GTMDs

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POETIC 7



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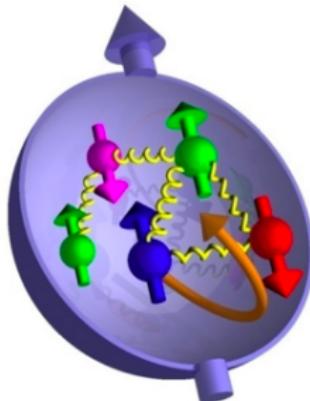
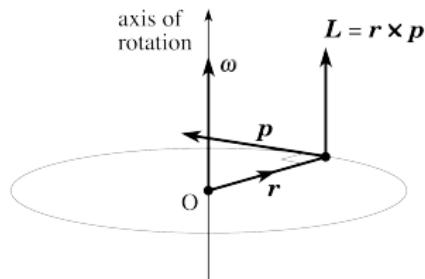
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Definition of OAM

- $L_i = \epsilon_{ijk} r_j p_k$
- So we need some info about r and p distributions.
- $[r_i, p_j] = i$
↳ Wigner Distributions probability quasidistribution.



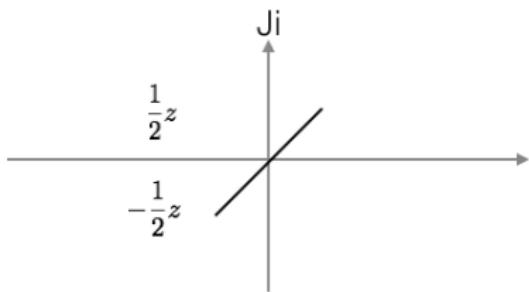
Parton Orbital Angular Momentum

The combined quark and gluon motion about the center of momentum of the system in the transverse plane.

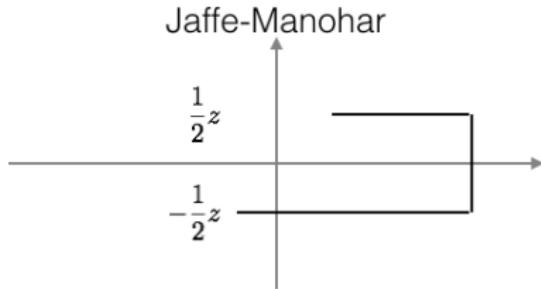
- Two different definitions of angular momentum from Ji and Jaffe-Manohar.

Ji vs. Jaffe-Manohar

- Straight gauge link versus staple gauge link
↳ natural gauge invariance in one versus the other.



$$\mathcal{L}_{Ji} = i\vec{r} \times \vec{\mathcal{D}}$$



$$\mathcal{L}_{JM} = i\vec{r} \times \vec{\partial}$$

Ji Spin Decomposition

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + L_q + J_g$$

JM Spin Decomposition

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

GTMDs¹

Generalized Parton Correlation Function

$$W_{\Lambda,\Lambda'}^{\Gamma} = \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \langle p', \Lambda' | \bar{\psi} \left(-\frac{1}{2} z \right) \Gamma \mathcal{U} \psi \left(\frac{1}{2} z \right) | p, \Lambda \rangle$$

Generalized Transverse Momentum Distribution Function (GTMD)

$$W_{\Lambda,\Lambda'}^{\Gamma} = \int \frac{dz^- d\vec{z}^T}{(2\pi)^3} e^{ixp^+ z^- - i\vec{k}^T \cdot \vec{z}^T} \langle p', \Lambda' | \bar{\psi}^q \left(-\frac{1}{2} z \right) \Gamma \mathcal{U} \psi^q \left(\frac{1}{2} z \right) | p, \Lambda \rangle \Big|_{z^+=0}$$

$$W_{\Lambda,\Lambda'}^{\gamma^+} = \frac{1}{2M} \bar{u}(P', \Lambda') \left[F_{1,1} + \frac{i\sigma^{i+} k_T^i}{P^+} F_{1,2} + \frac{i\sigma^{i+} \Delta_T^i}{P^+} F_{1,3} + \frac{i\sigma^{ij} k_T^i \Delta_T^j}{M^2} F_{1,4} \right] u(P, \Lambda)$$

GTMD Parametrization

$$W_{\Lambda,\Lambda'}^{\gamma^+} = \left(F_{1,1} + i\Lambda \frac{k_T \times \Delta_T}{M^2} F_{1,4} \right) \delta_{\Lambda,\Lambda'} + \left(\frac{\Lambda \Delta^1 + i\Delta^2}{2M} (2F_{1,3} - F_{1,1}) + \frac{\Lambda k^1 + ik^2}{M} F_{1,2} \right) \delta_{\Lambda,-\Lambda'}$$

¹Meißner, Metz, Schlegel arxiv: 0906.5323

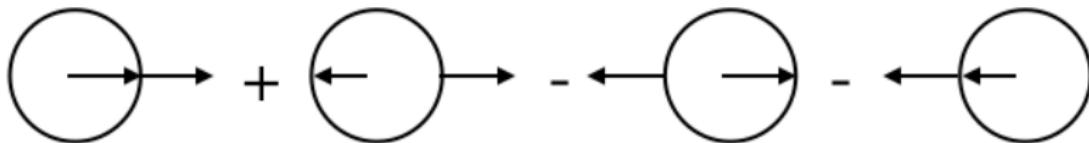
GTMDs and Helicity Amplitudes

- We can use the familiar projection operator and look at the quark non-flip helicity amplitudes.
- $\Gamma = \gamma^+ (1 \pm \gamma^5)$

Helicity Amplitudes

$$A_{\Lambda, \pm, \Lambda', \pm} = \int \frac{dz^- d\bar{z}^T}{(2\pi)^3} e^{ixp^+ z^- - i\vec{k}^T \cdot \bar{z}^T} \langle p', \Lambda' | \bar{\psi}^q \left(-\frac{1}{2} z \right) \gamma^+ (1 \pm \gamma^5) \psi^q \left(\frac{1}{2} z \right) | p, \Lambda \rangle$$

$$\frac{4i}{M^2} (\vec{k}_T \times \vec{\Delta}_T) F_{1,4} = A_{++++} + A_{+-+-} - A_{-+-+} - A_{----}$$



$F_{1,4}$ and quark OAM ²

- So how do we tie $F_{1,4}$ to quark OAM? $F_{1,4}$ is the coefficient of a Δ_T dependent term which is the Fourier Transform of the transverse position from the center of momentum, b_T .

$$\mathcal{L}_q = \int dx d^2 k_T d^2 b_T (b_T \times k_T) \hat{W}_{\Lambda, \Lambda'}^{\gamma^+} \delta_{\Lambda, \Lambda'}$$

$$\hat{W}_{\Lambda, \Lambda'}^{\gamma^+} \delta_{\Lambda, \Lambda'} = \mathcal{F}_{1,1} - \frac{1}{M^2} (k_T \times \partial_{b_T}) \mathcal{F}_{1,4}$$

- The $F_{1,1}$ term is 0 so there is no quark OAM in an unpolarized nucleon. For the second term we find the relation:

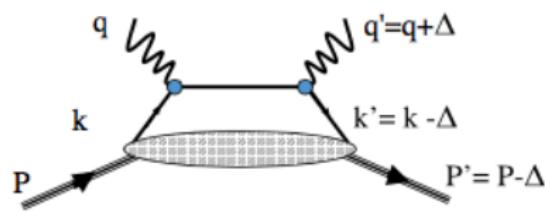
$F_{1,4}$ and OAM relation

$$\mathcal{L}_q = - \int dx d^2 k_T \frac{k_T^2}{M^2} F_{1,4}$$

²Lorcé and Pasquini arxiv: 1106.0139v1

Kinematics

- To get anywhere we first need to define some kinematics



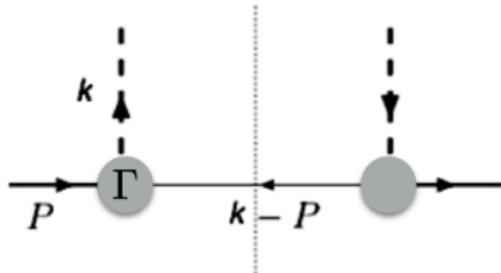
$$P = \left[P^+, \quad \frac{M^2}{2P^+}, \quad 0 \right]^T$$

$$\Delta = \left[0, \quad \frac{-\Delta_T^2}{2P^+}, \quad \Delta^T \right]^T$$

$$P' = \left[P^+, \quad \frac{M^2 + \Delta_T^2}{2P^+}, \quad -\Delta^T \right]^T$$

- Utilizing lightcone coordinates: $a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}$ and a^T
- Δ is the momentum transfer, x is defined to be the fraction of the longitudinal momentum carried by the quark from the proton, and we define the skewness $\xi = 0$.

Ji Spectator Model Calculation



- We can model our quark-diquark-proton vertex:

$$\Gamma = g_s \frac{k^2 - m^2}{(k^2 - M_\Lambda^2)^2}$$

- Our vertex functions that we will use to calculate our GTMD can be modeled in this case as a simple overlap of Dirac Spinors

$$\phi_{\Lambda,\lambda}(k, P) = \Gamma(k) \frac{\bar{u}(k, \lambda) U(P, \Lambda)}{k^2 - m^2}$$

- Our helicity amplitudes are then the interference between the two vertex functions for each side.

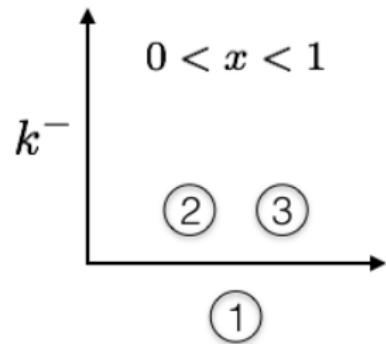
$$A_{\Lambda,\lambda,\Lambda',\lambda'} = \phi_{\Lambda',\lambda'}^*(k', P') \phi_{\Lambda,\lambda}(k, P)$$

Pole Structure and the Cutkovsky Rules

$$\kappa_1^- = -\frac{M_X^2 + k_T^2 - (1-x)M^2 - i\epsilon}{2(1-x)}$$

$$\kappa_2^- = \frac{m^2 + k_T^2 - i\epsilon}{2x}$$

$$\kappa_3^- = \frac{m^2 + (k^T - \Delta^T)^2 - x\Delta_T^2 - i\epsilon}{2x}$$



Here we directly see that the diquark is on mass shell.

$$k = \left[xP^+, \quad \frac{M^2}{2P^+} - \frac{M_X^2 + k_T^2}{2P^+(1-x)}, \quad k_T \right]^T$$

Ji Vertex Functions

- Using our defined coordinates, we evaluate our vertex functions which can be used to evaluate any number of helicity amplitudes.

Tree Level Vertex Functions

$$\phi_{++}(k, P) = \frac{1}{\sqrt{x}} g_s \frac{xM + m}{(k^2 - M_\Lambda^2)^2}$$

$$\phi_{+-}(k, P) = -\frac{1}{\sqrt{x}} g_s \frac{k^1 + ik^2}{(k^2 - M_\Lambda^2)^2}$$

$$\phi_{-+}(k, P) = -\frac{1}{\sqrt{x}} g_s \frac{-k^1 + ik^2}{(k^2 - M_\Lambda^2)^2}$$

$$\phi_{--}(k, P) = \frac{1}{\sqrt{x}} g_s \frac{xM + m}{(k^2 - M_\Lambda^2)^2}$$

$$\phi_{++}(k', P') = \frac{1}{\sqrt{x}} g_s \frac{xM + m}{((k - \Delta)^2 - M_\Lambda^2)^2}$$

$$\phi_{+-}(k', P') = -\frac{1}{\sqrt{x}} g_s \frac{\tilde{k}^1 + i\tilde{k}^2}{((k - \Delta)^2 - M_\Lambda^2)^2}$$

$$\phi_{-+}(k', P') = -\frac{1}{\sqrt{x}} g_s \frac{-\tilde{k}^1 + i\tilde{k}^2}{((k - \Delta)^2 - M_\Lambda^2)^2}$$

$$\phi_{--}(k', P') = \frac{1}{\sqrt{x}} g_s \frac{xM + m}{((k - \Delta)^2 - M_\Lambda^2)^2}$$

Ji Helicity Amplitudes

Unintegrated Tree Level Helicity Amplitudes for $F_{1,4}$

$$A_{++++} = \frac{1}{x} g_s^2 \frac{(xM + m)^2}{((k - \Delta)^2 - M_\Lambda^2)^2 (k^2 - M_\Lambda^2)^2}$$

$$A_{+-+-} = \frac{1}{x} g_s^2 \frac{(\tilde{k}^1 - i\tilde{k}^2)(k^1 + ik^2)}{((k - \Delta)^2 - M_\Lambda^2)^2 (k^2 - M_\Lambda^2)^2}$$

$$A_{-+-+} = \frac{1}{x} g_s^2 \frac{(\tilde{k}^1 + ik^2)(k^1 - ik^2)}{((k - \Delta)^2 - M_\Lambda^2)^2 (k^2 - M_\Lambda^2)^2}$$

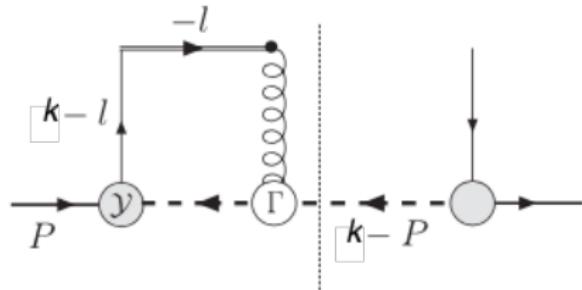
$$A_{----} = \frac{1}{x} g_s^2 \frac{(xM + m)^2}{((k - \Delta)^2 - M_\Lambda^2)^2 (k^2 - M_\Lambda^2)^2}$$

$$F_{1,4}^{(1)} = \int dk_T k_T^3 \frac{g_s^2 2\pi (1-x)^4 (\Lambda(x) - k_T^2 - (1-x)^2 \Delta_T^2)}{2(\Lambda(x) - k_T^2)^2 [(\Lambda(x) - k_T^2 - (1-x)^2 \Delta_T^2)^2 - 4(1-x)^2 k_T^2 \Delta_T^2]^{\frac{3}{2}}}$$

$$\Lambda(x) = x(1-x)M^2 - xM_X^2 - (1-x)M_\Lambda^2$$

Jaffe-Manohar Spectator Model Calculation

To simulate the staple gauge link, we introduce a gluon exchange on one side of the diagram and find the interference with a non gluon exchange.³



$$\begin{array}{c} \bullet \\ \circlearrowleft \\ \rho \end{array} = -ie_c n_-^\rho$$

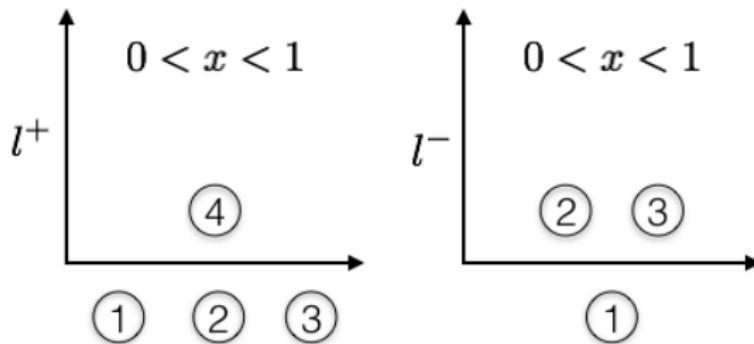
$$\begin{array}{c} (-l) \\ \longrightarrow \end{array} = \frac{i}{-l^+ + i\epsilon}$$

$$\phi_{\lambda, \Lambda}^g = \int \frac{d^4 I}{(2\pi)^4} \frac{1}{(P - k + I)^2 - M_X^2 + i\epsilon} \Gamma_s^\rho \frac{1}{I^2 - m_g^2 + i\epsilon} - ie_c n_-^\rho \frac{-i}{I^+ - i\epsilon} \frac{\bar{u}(k - I, \lambda)}{(k - I)^2 - m^2 + i\epsilon} ig_s(k - I) U(P, \Lambda)$$

$$\Gamma_s^\rho = ie_c (2P - 2k + I)^\rho$$

³Bacchetta, Conti, Radici PRD 78, 074010 (2008)

Pole Structure and the Cutkovsky Rules



Here we directly see that the diquark and eikonalized quark are on mass shell.

$$l = \left[0, \quad \frac{l_T^2 - 2k_T \cdot l_T}{2P^+(1-x)}, \quad l_T \right]^T$$

JM Vertex Functions

- The same versatility from the gluon exchange vertex functions. ↴ a promise of more interesting calculations to come.

Gluon Exchange Vertex Functions

$$\phi_{++}^g = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s e_c^2 2P^+ (1-x)(m+xM)}{\sqrt{x}(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2}$$

$$\phi_{+-}^g = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s e_c^2 2P^+ (1-x)(l^1 + il^2 - ik^2 - k^1)}{\sqrt{x}(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2}$$

$$\phi_{-+}^g = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s e_c^2 2P^+ (1-x)(-l^1 + il^2 - ik^2 + k^1)}{\sqrt{x}(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2}$$

$$\phi_{--}^g = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s e_c^2 2P^+ (1-x)(m+xM)}{\sqrt{x}(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2}$$

JM Helicity Amplitudes

Unintegrated Gluon Exchange Helicity Amplitudes

$$A_{++++} = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s^2 e_c^2 2P^+ (1-x)(m+xM)^2}{x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

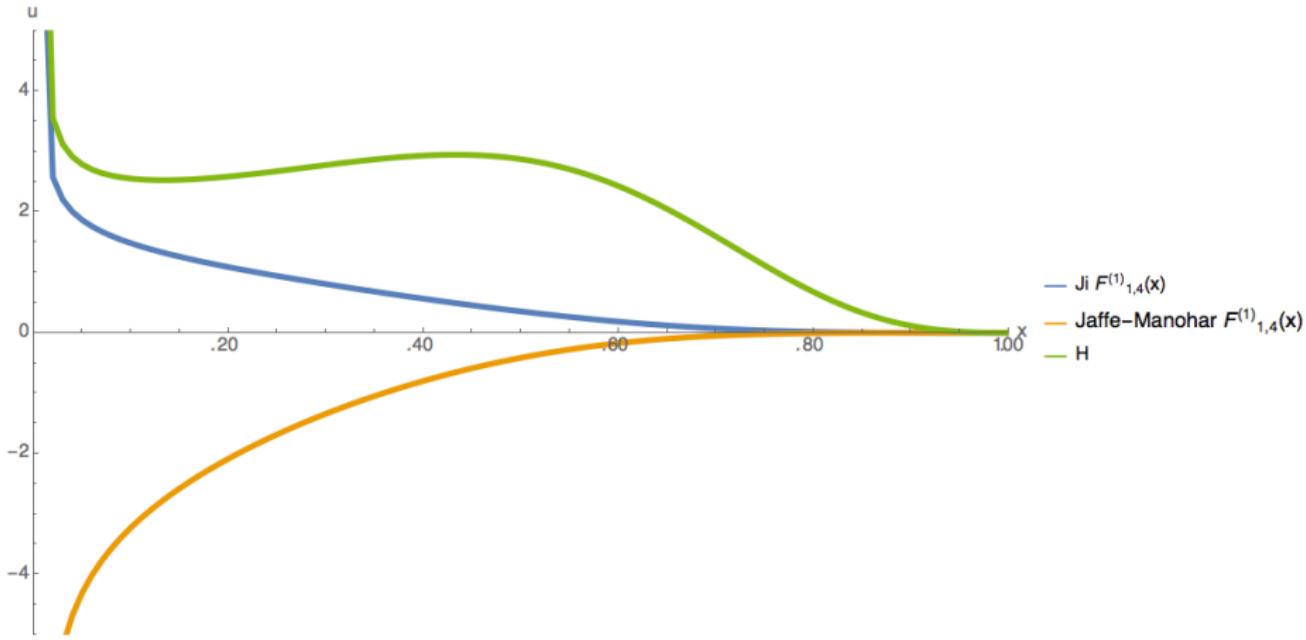
$$A_{+-+-} = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s^2 e_c^2 2P^+ (1-x)(\tilde{k}^1 - i\tilde{k}^2)(l^1 + il^2 - ik^2 - k^1)}{x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

$$A_{-+-+} = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s^2 e_c^2 2P^+ (1-x)(-i\tilde{k}^2 - \tilde{k}^1)(-l^1 + il^2 - ik^2 + k^1)}{x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

$$A_{----} = - \int \frac{d^2 l_T}{(2\pi)^2} \frac{g_s^2 e_c^2 2P^+ (1-x)(m+xM)^2}{x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

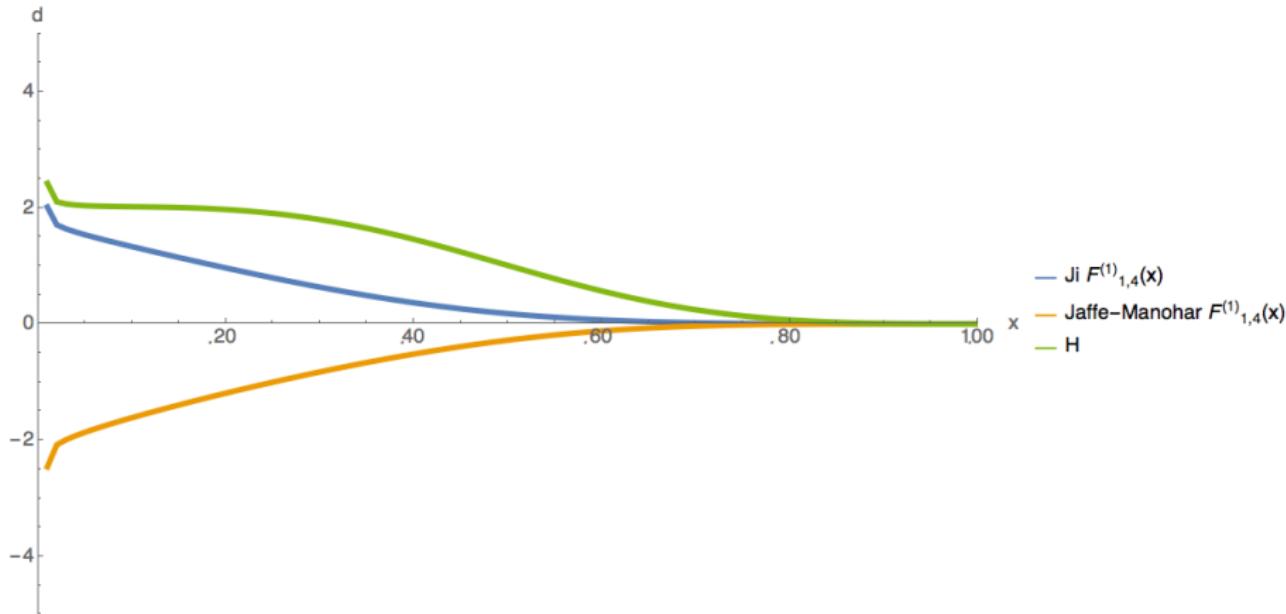
$$F_{1,4}^{(1),g} = \int dk_T dl_T \frac{g_s^2 e_c^2 (1-x)^5 k_T^3 (\Lambda(x) - k_T^2 - (1-x)^2 \Delta_T^2) (\Lambda(x) - k_T^2 + l_T^2)}{2l_T [(\Lambda(x) - k_T^2 - (1-x)^2 \Delta_T^2)^2 - 2k_T^2 \Delta_T^2 (1-x)^2]^{\frac{3}{2}} [(\Lambda(x) - k_T^2 - l_T^2)^2 - 4k_T^2 l_T^2]^{\frac{3}{2}}}$$

u-quarks



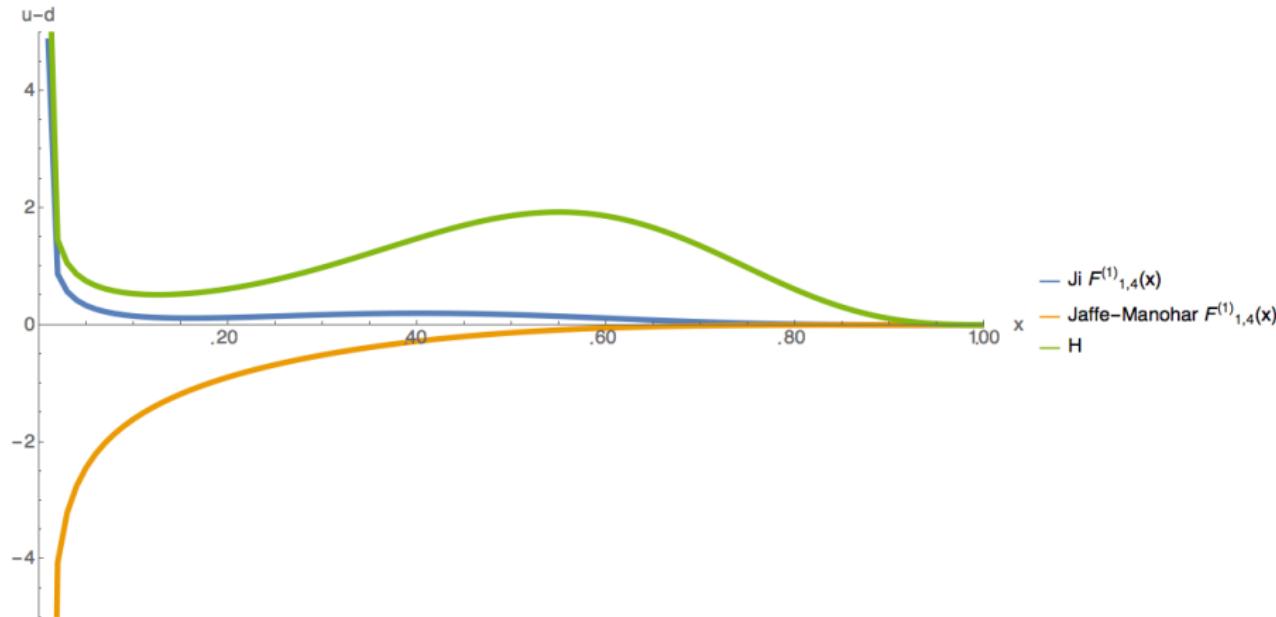
$\int dxH$	$\int dxF^{(1),Ji}_{1,4}$	$\int dxF^{(1),JM}_{1,4}$
2.01	0.57	-1.08

d-quarks



$\int dx H$	$\int dx F_{1,4}^{(1), Ji}$	$\int dx F_{1,4}^{(1), JM}$
1.00	.43	-0.54

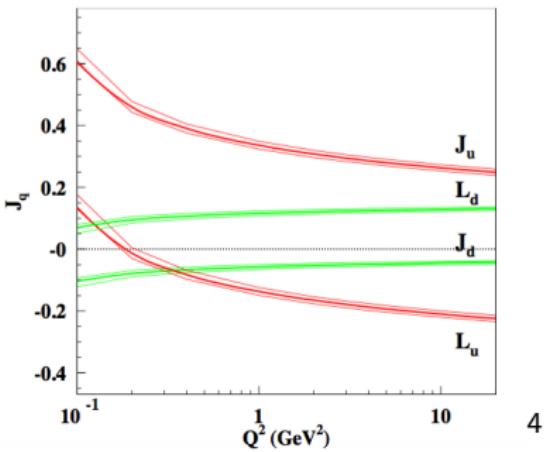
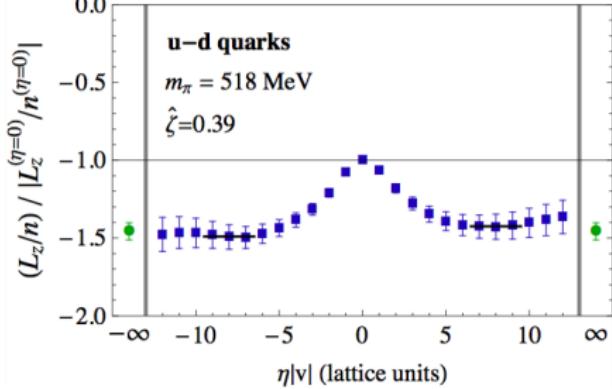
u-d



$\int dxH$	$\int dxF_{1,4}^{(1),Ji}$	$\int dxF_{1,4}^{(1),JM}$
1.00	0.15	-0.54

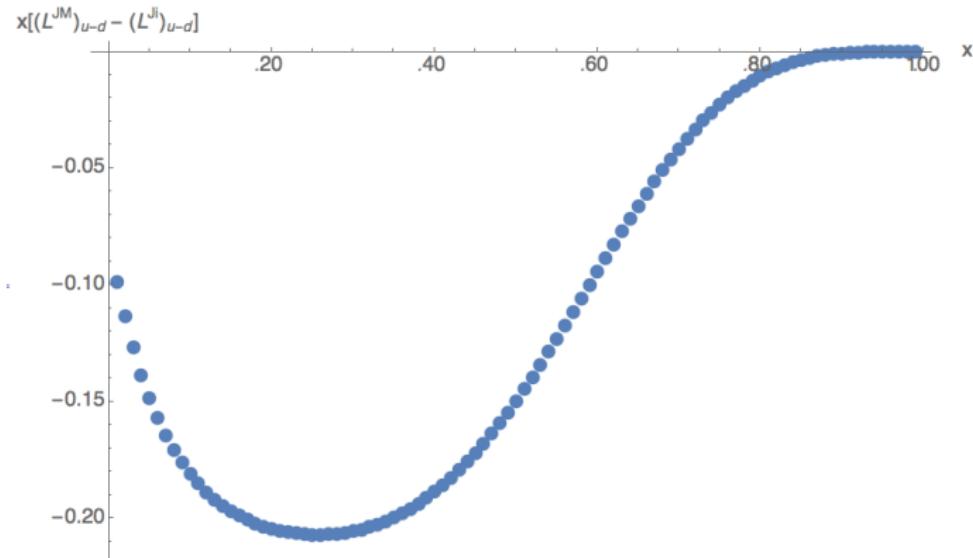
Comparison with the Lattice

Large effect calculated on the lattice (M. Engelhardt, preliminary)



⁴Gonzalez, Liuti, Goldstein, Kathuria arxiv: 1206.1876

Qiu-Sterman⁵ / Torque term⁶



$$\int dx x \left[(L^{JM})_{u-d} - (L^{Ji})_{u-d} \right] = -0.69$$

⁵Phys. Rev. Lett. 67, 2264 (1991)

⁶Matthias Burkardt arxiv:1205.2916v3

Summary

- $F_{1,4}^{(1)}(x)$ via Ji and Jaffe-Manohar in diquark model.
 - ↳ Relative size compared to experimentally measured GPDs.
- Extension to $\Delta \neq 0$ and $\xi \neq 0$
- Q^2 evolution to compare with lattice measurements.
- Calculate f_{1T}^\perp via $F_{1,2}$
- Verification of Twist 3 GPD sum rule from Abha.
- Qiu-Sterman term
 - ↳ How big is it?